



Numerische Mathematik: Hausaufgabe 2

Aufgabe 1

Aufg. 1-1) $f(x) = (x-1)^8$ $f'(x) = 8(x-1)^7$

mit Startwert 1,1 $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

i	0	1	2	3	4
x_{i+1}	1,1	1,0875	1,0765625	1,066992	1,058618

⇒ Ergebnisse ändern sich noch, deshalb mit Iterationen notwendig.

Aufg. 1-2) $f(x) = (x-1)^{20}$ $f'(x) = 20(x-1)^{19}$

mit Startwert 1,1 $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

i	0	1	2	3	4
x_{i+1}	1,1	1,095	1,090	1,085	1,081

⇒ Die quadratische Konvergenz wird nicht erreicht. Die Werte ändern sich mit jeder Iteration deutlich. Mehr Iterationsschritte sind erforderlich.

Aufg. 1-3) $f(x) = \frac{1}{2} + \frac{1}{4}x^2 - x \cdot \sin(x) - \frac{1}{2}\cos(2x)$

$f'(x) = \frac{1}{2} \cdot x - x \cdot \cos(x) - \sin(x) + \sin(2x)$

mit Startwert $\frac{\pi}{2}$ und $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

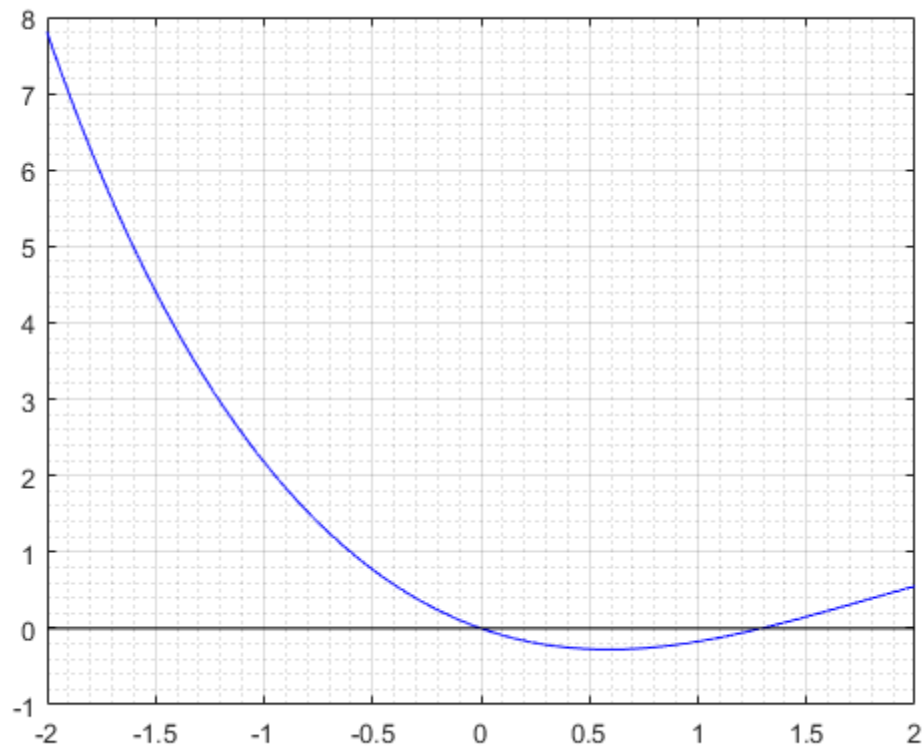
i	0	1	2	3	4	5	6	7	8	9	10
x_{i+1}	$\frac{\pi}{2}$	1,785	1,846	1,8708	1,8833	1,8895	1,8925	1,8940	1,8947	1,8951	1,8953

⇒ Für eine Genauigkeit von 10^{-3} sind 9. Iterationen notwendig, was für das Newton-Verfahren ungewöhnlich ist.



Aufgabe 2

Die Funktion wird in allen Skripten zunächst geplottet, um die Nullstellen zu erkennen:



a)

Quelltext siehe *G08_H2_Aufgabe_2a.pdf*

Variables - aTABELLE				
5x4 string				
BERECHNUNG	1	2	3	4
aTABELLE				
1 i		X_i	X_{i+1}	relativer Fehler
2 0		1.35000000000000010...	1.29384574638095180...	0.043401
3 1		1.29384574638095180...	1.29269624644439850...	0.00088923
4 2		1.29269624644439850...	1.29269571937350940...	4.0773e-07
5 3		1.29269571937350940...	1.29269571937339830...	8.5884e-14

Startwert wurde bei 1,35 gewählt, was dem mittleren Wert des Intervalls entspricht.



b)

Quelltext siehe G08_H2_Aufgabe_2b.pdf

Variables - aTABELLE						
41x6 string						
aTABELLE	1	2	3	4	5	6
1		Xu	Xo	Xm	f(Xm)	f(Xu)*f(Xm)
2	0	1.10000000000000010...	1.60000000000000010...	1.35000000000000010...	0.04023357355285001...	<0
3	1	1.10000000000000010...	1.35000000000000010...	1.22500000000000010...	-0.0451882732125844...	>0
4	2	1.22500000000000010...	1.35000000000000010...	1.28750000000000010...	-0.0035622732266422...	>0
5	3	1.28750000000000010...	1.35000000000000010...	1.31875000000000010...	0.01808327855461017...	<0
6	4	1.28750000000000010...	1.31875000000000010...	1.30312500000000010...	0.00719505246683549...	<0
7	5	1.28750000000000010...	1.30312500000000010...	1.29531250000000010...	0.00179973236771513...	<0
8	6	1.28750000000000010...	1.29531250000000010...	1.29140625000000010...	-0.0008854715813622...	>0
9	7	1.29140625000000010...	1.29531250000000010...	1.29335937500000010...	0.00045608471006125...	<0
10	8	1.29140625000000010...	1.29335937500000010...	1.29238281250000010...	-0.0002149554320369...	>0
11	9	1.29238281250000010...	1.29335937500000010...	1.29287109375000010...	0.00012049921186646...	<0
12	10	1.29238281250000010...	1.29287109375000010...	1.29262695312500010...	-0.0000472444758655...	>0
13	11	1.29262695312500010...	1.29287109375000010...	1.29274902343750010...	0.00003662327767967...	<0
14	12	1.29262695312500010...	1.29274902343750010...	1.29268798828125010...	-0.0000053116218136...	>0
15	13	1.29268798828125010...	1.29274902343750010...	1.29271850585937510...	0.00001565557227040...	<0
16	14	1.29268798828125010...	1.29271850585937510...	1.29270324707031260...	0.00000517191131049...	<0
17	15	1.29268798828125010...	1.29270324707031260...	1.29269561767578130...	-0.000000698712313...	>0
18	16	1.29269561767578130...	1.29270324707031260...	1.29269943237304700...	0.00000255101604468...	<0
19	17	1.29269561767578130...	1.29269943237304700...	1.29269752502441420...	0.00000124057140793...	<0
20	18	1.29269561767578130...	1.29269752502441420...	1.29269657135009770...	0.00000058534983859...	<0
21	19	1.29269561767578130...	1.29269657135009770...	1.29269609451293950...	0.00000025773924122...	<0
22	20	1.29269561767578130...	1.29269609451293950...	1.29269585609436040...	0.00000009393398936...	<0
23	21	1.29269561767578130...	1.29269585609436040...	1.29269573688507090...	0.00000001203137511...	<0
24	22	1.29269561767578130...	1.29269573688507090...	1.29269567728042610...	-0.0000000289199290...	>0
25	23	1.29269567728042610...	1.29269573688507090...	1.29269570708274850...	-0.0000000084442772...	>0
26	24	1.29269570708274850...	1.29269573688507090...	1.29269572198390970...	0.00000000179354886...	<0
27	25	1.29269570708274850...	1.29269572198390970...	1.29269571453332910...	-0.0000000033253641...	>0
28	26	1.29269571453332910...	1.29269572198390970...	1.29269571825861940...	-0.0000000007659076...	>0
29	27	1.29269571825861940...	1.29269572198390970...	1.29269572012126450...	0.00000000051382059...	<0
30	28	1.29269571825861940...	1.29269572012126450...	1.29269571918994200...	-0.0000000001260435...	>0
31	29	1.29269571918994200...	1.29269572012126450...	1.29269571965560330...	0.00000000019388851...	<0
32	30	1.29269571918994200...	1.29269571965560330...	1.29269571942277260...	0.00000000003392253...	<0
33	31	1.29269571918994200...	1.29269571942277260...	1.29269571930635730...	-0.000000000460604...	>0
34	32	1.29269571930635730...	1.29269571942277260...	1.29269571936456500...	-0.0000000000060689...	>0
35	33	1.29269571936456500...	1.29269571942277260...	1.29269571939366880...	0.00000000001392674...	<0
36	34	1.29269571936456500...	1.29269571939366880...	1.29269571937911690...	0.00000000000392885...	<0
37	35	1.29269571936456500...	1.29269571937911690...	1.29269571937184090...	-0.0000000000010700...	>0
38	36	1.29269571937184090...	1.29269571937911690...	1.29269571937547890...	0.00000000000142941...	<0
39	37	1.29269571937184090...	1.29269571937547890...	1.29269571937365990...	0.00000000000017968...	<0
40	38	1.29269571937184090...	1.29269571937365990...	1.29269571937275040...	-0.0000000000004451...	>0
41	39	1.29269571937275040...	1.29269571937365990...	1.29269571937320520...	-0.0000000000001327...	>0

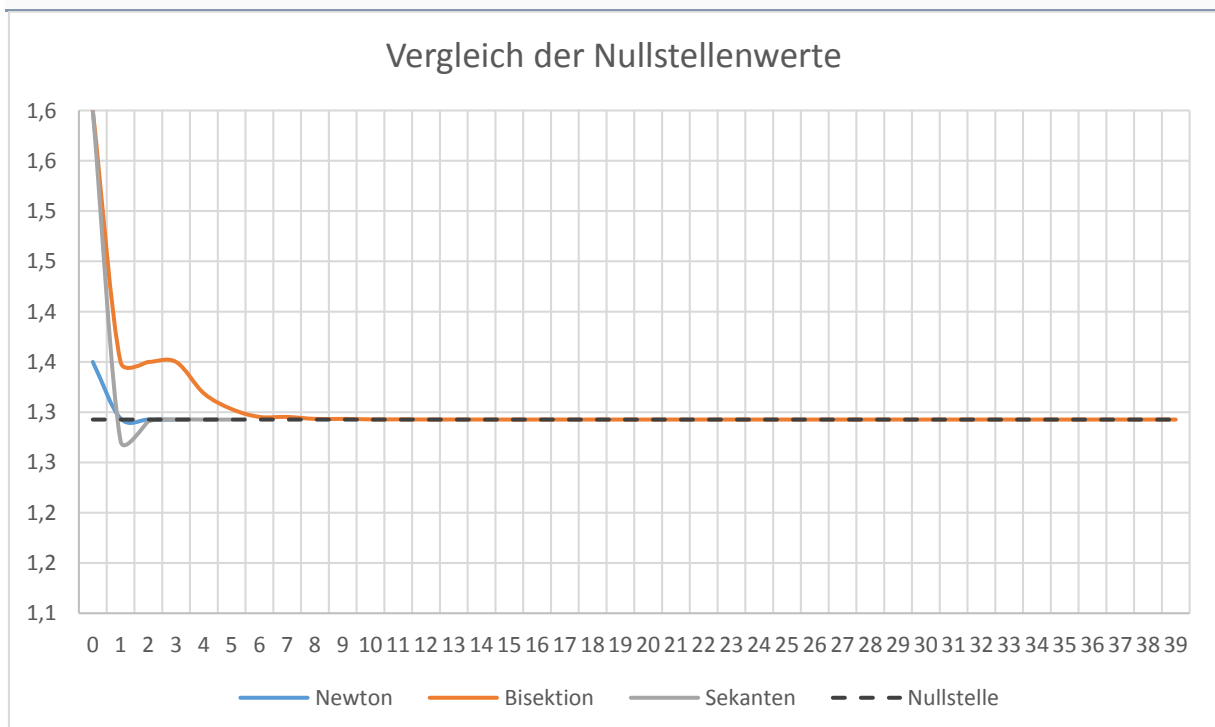


c)

Quelltext siehe *G08_H2_Aufgabe_2b.pdf*

Variables - aTABELLE				
str 7x4 string				
aTABELLE				
	1	2	3	4
1	i	X_{i-1}	X_i	X_{i+1}
2	0	1.10000000000000010...	1.60000000000000010...	1.27157163863765700...
3	1	1.60000000000000010...	1.27157163863765700...	1.29082211364092240...
4	2	1.27157163863765700...	1.29082211364092240...	1.29271195849806660...
5	3	1.29082211364092240...	1.29271195849806660...	1.29269570718994590...
6	4	1.29271195849806660...	1.29269570718994590...	1.29269571937331930...
7	5	1.29269570718994590...	1.29269571937331930...	1.29269571937339830...

d)



Der Plot zeigt auf der y-Achse den angenäherten Wert der Nullstelle über die Iterationsschritte auf der x-Achse.

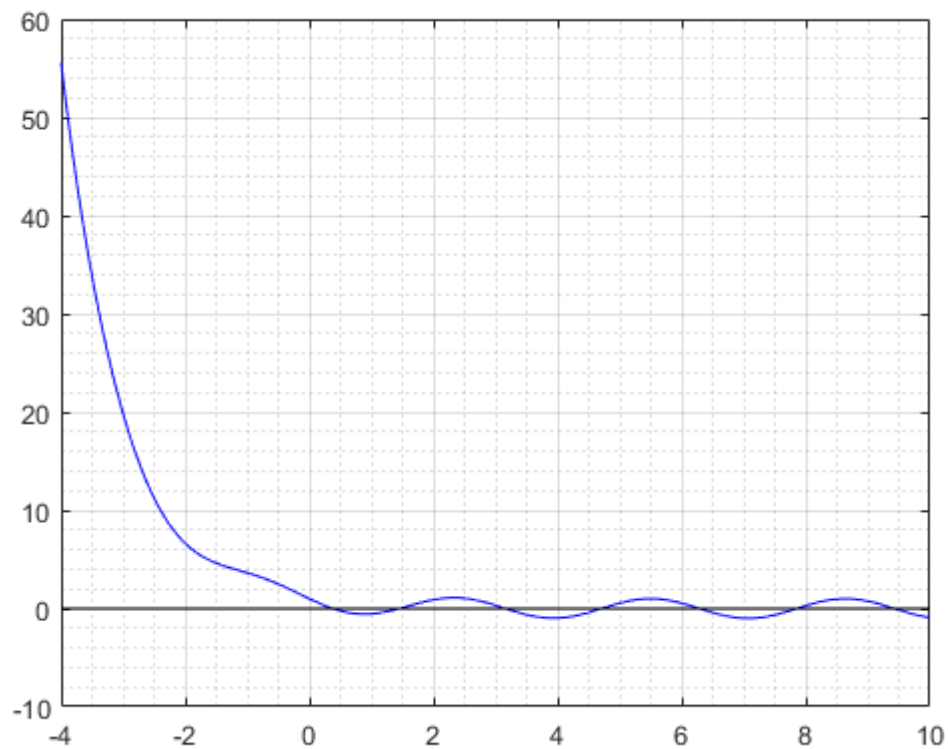
Man erkennt, dass das Newton-Verfahren mit dem gewählten Startwert, dicht gefolgt vom Sekanten-Verfahren, am schnellsten konvergiert. Das Bisektions-Verfahren ist dagegen träge in der Konvergenz.



Aufgabe 3

a)

Quelltext siehe *G08_H2_Aufgabe_3a.pdf*



Da ein sinusförmiger Auslauf der Funktion zu erkennen ist, wird das Verfahren immer die Nullstelle finden, die sich am nächsten am Startwert befindet.

Startwert: 0,5

Variables - aTABELLE				
aTABELLE				
	1	2	3	4
1 i		X_i	X_{i+1}	Abweichung
2 0		0.5000000000000000...	0.36074600592429307...	0.13925
3 1		0.36074600592429307...	0.37741438801916721...	0.0083342
4 2		0.37741438801916721...	0.37763280390682930...	5.4604e-05
5 3		0.37763280390682930...	0.37763284207127579...	4.7706e-09



Startwert: 1,5

Variables - aTABELLE				
str 5x4 string				
aTABELLE				
	1	2	3	4
1 i		X_i	X_{i+1}	Abweichung
2 0		1.5000000000000000...	1.45331990410040120...	0.04668
3 1		1.45331990410040120...	1.45273805035070950...	0.00029093
4 2		1.45273805035070950...	1.45273793491169090...	2.886e-08
5 3		1.45273793491169090...	1.45273793491168650...	5.5511e-16

Startwert: 2,5

Variables - aTABELLE				
str 14x4 string				
aTABELLE				
	1	2	3	4
1 i		X_i	X_{i+1}	Abweichung
2 0		2.5000000000000000...	4.10300932215965460...	1.603
3 1		4.10300932215965460...	5.47301627597271170...	0.685
4 2		5.47301627597271170...	-5.1019006820381518...	2.6437
5 3		-5.1019006820381518...	-4.0974773078995534...	0.12555
6 4		-4.0974773078995534...	-3.0704065243325056...	0.064192
7 5		-3.0704065243325056...	-2.1605730225181023...	0.028432
8 6		-2.1605730225181023...	-1.1810142598033995...	0.015306
9 7		-1.1810142598033995...	0.97709769591021223...	0.01686
10 8		0.97709769591021223...	2.45927574675950170...	0.0057898
11 9		2.45927574675950170...	4.60982123221871460...	0.0042003
12 10		4.60982123221871460...	4.70927449850607170...	9.7122e-05
13 11		4.70927449850607170...	4.70787697992781330...	6.8238e-07
14 12		4.70787697992781330...	4.70787696145673620...	4.5095e-12

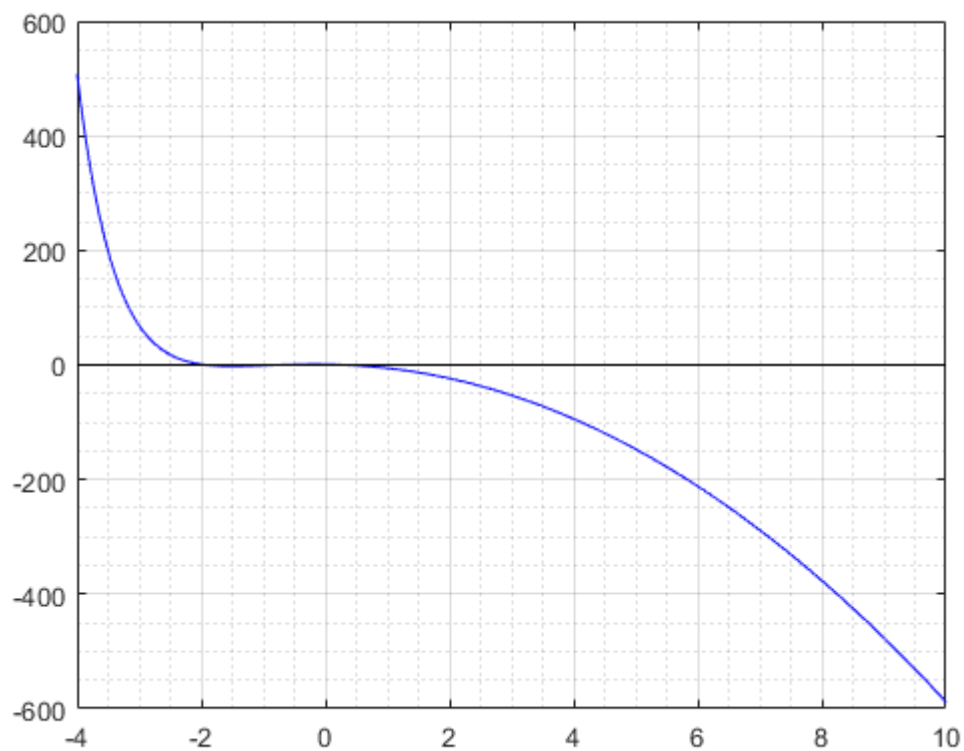
Startwert: π



Variables - aTABELLE				
str 4x4 string				
aTABELLE				
	1	2	3	4
1 i		X_i	X_{i+1}	Abweichung
2 0		3.14159265358979310...	3.16274262612937610...	0.02115
3 1		3.16274262612937610...	3.16275351099395240...	5.4424e-06
4 2		3.16275351099395240...	3.16275351100009240...	1.535e-12

b)

Quelltext siehe *G08_H2_Aufgabe_3b.pdf*



Da die Startwerte rechtsseitig der dritten Nullstelle angesetzt sind, wird das Verfahren immer nur diese finden.



Startwert: 0,5

Variables - aTABELLE				
str 6x4 string				
aTABELLE				
	1	2	3	4
1 i		X_i	X_{i+1}	Abweichung
2 0		0.5000000000000000...	0.34532771198010442...	0.15467
3 1		0.34532771198010442...	0.32009570202478377...	0.012616
4 2		0.32009570202478377...	0.31940374033911878...	0.00017299
5 3		0.31940374033911878...	0.31940322124535570...	6.4887e-08
6 4		0.31940322124535570...	0.31940322124506360...	1.8256e-14

Startwert: 1,5

Variables - aTABELLE				
str 7x4 string				
aTABELLE				
	1	2	3	4
1 i		X_i	X_{i+1}	Abweichung
2 0		1.5000000000000000...	0.76122169402796236...	0.73878
3 1		0.76122169402796236...	0.43111227099520272...	0.16505
4 2		0.43111227099520272...	0.33044828569452900...	0.025166
5 3		0.33044828569452900...	0.31953259400877182...	0.0013645
6 4		0.31953259400877182...	0.31940323938379633...	8.0847e-06
7 5		0.31940323938379633...	0.31940322124506393...	5.6684e-10

Startwert: 2,5

Variables - aTABELLE				
str 8x4 string				
aTABELLE				
	1	2	3	4
1 i		X_i	X_{i+1}	Abweichung
2 0		2.5000000000000000...	1.25186708169373070...	1.2481
3 1		1.25186708169373070...	0.64401276774923744...	0.30393
4 2		0.64401276774923744...	0.38839586785996855...	0.063904
5 3		0.38839586785996855...	0.32393462737030249...	0.0080577
6 4		0.32393462737030249...	0.31942527925115244...	0.00028183
7 5		0.31942527925115244...	0.31940322177247232...	6.893e-07
8 6		0.31940322177247232...	0.31940322124506360...	8.2408e-12



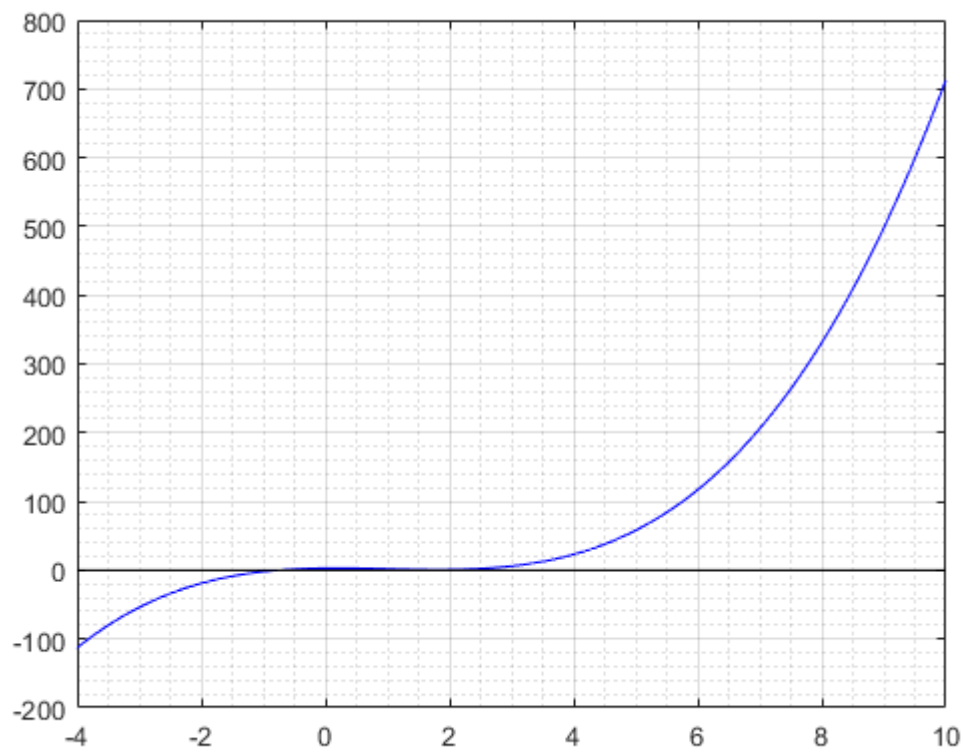
Startwert: 3,5

Variables - aTABELLE				
str 8x4 string				
aTABELLE				
	1	2	3	4
1	i	X_i	X_{i+1}	Abweichung
2	0	3.5000000000000000...	1.75034134748572060...	1.7497
3	1	1.75034134748572060...	0.88222445480886402...	0.43406
4	2	0.88222445480886402...	0.47973801327660159...	0.10062
5	3	0.47973801327660159...	0.34046914600880701...	0.017409
6	4	0.34046914600880701...	0.31986474847786645...	0.0012878
7	5	0.31986474847786645...	0.31940345193382125...	1.4416e-05
8	6	0.31940345193382125...	0.31940322124512127...	3.6045e-09



c)

Quelltext siehe *G08_H2_Aufgabe_3c.pdf*



Startwert: -1,0

Variables - aTABELLE				
str 6x4 string				
aTABELLE				
	1	2	3	4
1 i		X_i	X_{i+1}	Abweichung
2 0		-1.0000000000000000...	-0.8000000000000000...	0.2
3 1		-0.8000000000000000...	-0.7699481865284973...	0.015026
4 2		-0.7699481865284973...	-0.7692926629059410...	0.00016388
5 3		-0.7692926629059410...	-0.7692923542386997...	3.8583e-08
6 4		-0.7692923542386997...	-0.7692923542386314...	4.2744e-15



Startwert: 1,0

Variables - aTABELLE				
str 29x4 string				
aTABELLE	1	2	3	4
1 i		X_i	X_{i+1}	Abweichung
2 0		1.0000000000000000...	2.0000000000000000...	1
3 1		2.0000000000000000...	1.0000000000000000...	0.5
4 2		1.0000000000000000...	2.0000000000000000...	0.25
5 3		2.0000000000000000...	1.0000000000000000...	0.125
6 4		1.0000000000000000...	2.0000000000000000...	0.0625
7 5		2.0000000000000000...	1.0000000000000000...	0.03125
8 6		1.0000000000000000...	2.0000000000000000...	0.015625
9 7		2.0000000000000000...	1.0000000000000000...	0.0078125
10 8		1.0000000000000000...	2.0000000000000000...	0.0039063
11 9		2.0000000000000000...	1.0000000000000000...	0.0019531
12 10		1.0000000000000000...	2.0000000000000000...	0.00097656
13 11		2.0000000000000000...	1.0000000000000000...	0.00048828
14 12		1.0000000000000000...	2.0000000000000000...	0.00024414
15 13		2.0000000000000000...	1.0000000000000000...	0.00012207
16 14		1.0000000000000000...	2.0000000000000000...	6.1035e-05
17 15		2.0000000000000000...	1.0000000000000000...	3.0518e-05
18 16		1.0000000000000000...	2.0000000000000000...	1.5259e-05
19 17		2.0000000000000000...	1.0000000000000000...	7.6294e-06
20 18		1.0000000000000000...	2.0000000000000000...	3.8147e-06
21 19		2.0000000000000000...	1.0000000000000000...	1.9073e-06
22 20		1.0000000000000000...	2.0000000000000000...	9.5367e-07
23 21		2.0000000000000000...	1.0000000000000000...	4.7684e-07
24 22		1.0000000000000000...	2.0000000000000000...	2.3842e-07
25 23		2.0000000000000000...	1.0000000000000000...	1.1921e-07
26 24		1.0000000000000000...	2.0000000000000000...	5.9605e-08
27 25		2.0000000000000000...	1.0000000000000000...	2.9802e-08
28 26		1.0000000000000000...	2.0000000000000000...	1.4901e-08
29 27		2.0000000000000000...	1.0000000000000000...	7.4506e-09



Startwert: 2,0

Variables - aTABELLE				
str 29x4 string				
aTABELLE				
	1	2	3	4
1 i		X_i	X_{i+1}	Abweichung
2 0		2.0000000000000000...	1.0000000000000000...	1
3 1		1.0000000000000000...	2.0000000000000000...	0.5
4 2		2.0000000000000000...	1.0000000000000000...	0.25
5 3		1.0000000000000000...	2.0000000000000000...	0.125
6 4		2.0000000000000000...	1.0000000000000000...	0.0625
7 5		1.0000000000000000...	2.0000000000000000...	0.03125
8 6		2.0000000000000000...	1.0000000000000000...	0.015625
9 7		1.0000000000000000...	2.0000000000000000...	0.0078125
10 8		2.0000000000000000...	1.0000000000000000...	0.0039063
11 9		1.0000000000000000...	2.0000000000000000...	0.0019531
12 10		2.0000000000000000...	1.0000000000000000...	0.00097656
13 11		1.0000000000000000...	2.0000000000000000...	0.00048828
14 12		2.0000000000000000...	1.0000000000000000...	0.00024414
15 13		1.0000000000000000...	2.0000000000000000...	0.00012207
16 14		2.0000000000000000...	1.0000000000000000...	6.1035e-05
17 15		1.0000000000000000...	2.0000000000000000...	3.0518e-05
18 16		2.0000000000000000...	1.0000000000000000...	1.5259e-05
19 17		1.0000000000000000...	2.0000000000000000...	7.6294e-06
20 18		2.0000000000000000...	1.0000000000000000...	3.8147e-06
21 19		1.0000000000000000...	2.0000000000000000...	1.9073e-06
22 20		2.0000000000000000...	1.0000000000000000...	9.5367e-07
23 21		1.0000000000000000...	2.0000000000000000...	4.7684e-07
24 22		2.0000000000000000...	1.0000000000000000...	2.3842e-07
25 23		1.0000000000000000...	2.0000000000000000...	1.1921e-07
26 24		2.0000000000000000...	1.0000000000000000...	5.9605e-08
27 25		1.0000000000000000...	2.0000000000000000...	2.9802e-08
28 26		2.0000000000000000...	1.0000000000000000...	1.4901e-08
29 27		1.0000000000000000...	2.0000000000000000...	7.4506e-09



Startwert: 3,0

Variables - aTABELLE				
str 10x4 string				
aTABELLE				
	1	2	3	4
1 i		X_i	X_{i+1}	Abweichung
2 0		3.0000000000000000...	2.3999999999999990...	0.6
3 1		2.3999999999999990...	1.89896907216494950...	0.25052
4 2		1.89896907216494950...	-0.2887793276654215...	0.54694
5 3		-0.2887793276654215...	-1.1057672990136456...	0.10212
6 4		-1.1057672990136456...	-0.8291999504602743...	0.017285
7 5		-0.8291999504602743...	-0.7717158120621090...	0.0017964
8 6		-0.7717158120621090...	-0.7692965611557901...	3.7801e-05
9 7		-0.7692965611557901...	-0.7692923542513410...	3.2866e-08
10 8		-0.7692923542513410...	-0.7692923542386314...	4.9647e-14



Startwert: 3.0		xi+1		Abweichung	
i	xi	3	2.4	0.6	
	1	2.189869072	2.5052	0.54694	
	2	1.898596072	2.388779328	0.10212	
	3	2.288779328	1.105767299	0.0017285	
	4	-1.105767299	-0.82919995	0.0017964	
	5	-0.82919995	-1.77115812	3.78E-05	
	6	-0.77115812	-0.769296561	3.29E-08	
	7	-0.769296561	-0.769292354	4.96E-14	
	8	-0.769292354			

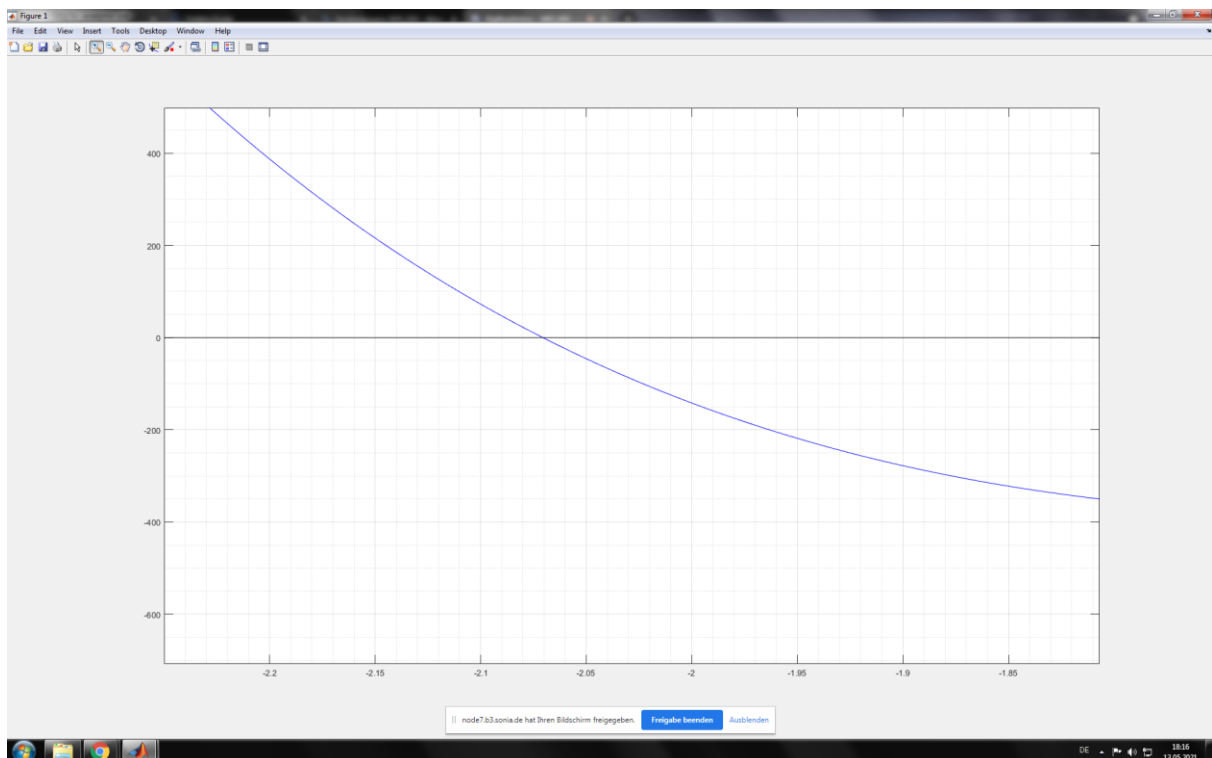
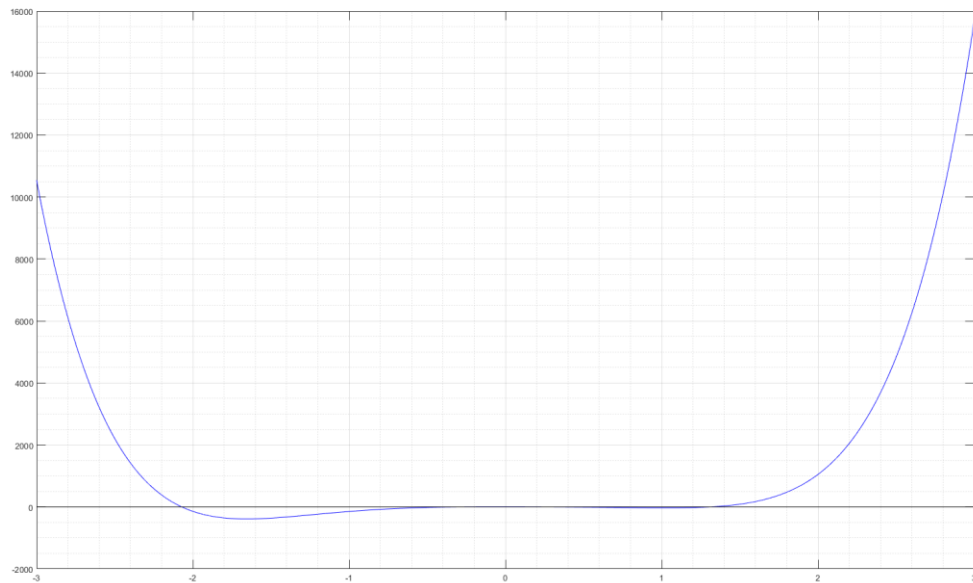
Der Startwert -1.0 liegt näher an der Nullstelle als der Startwert 3.0. Deswegen sind weniger Iterationsschritte nötig

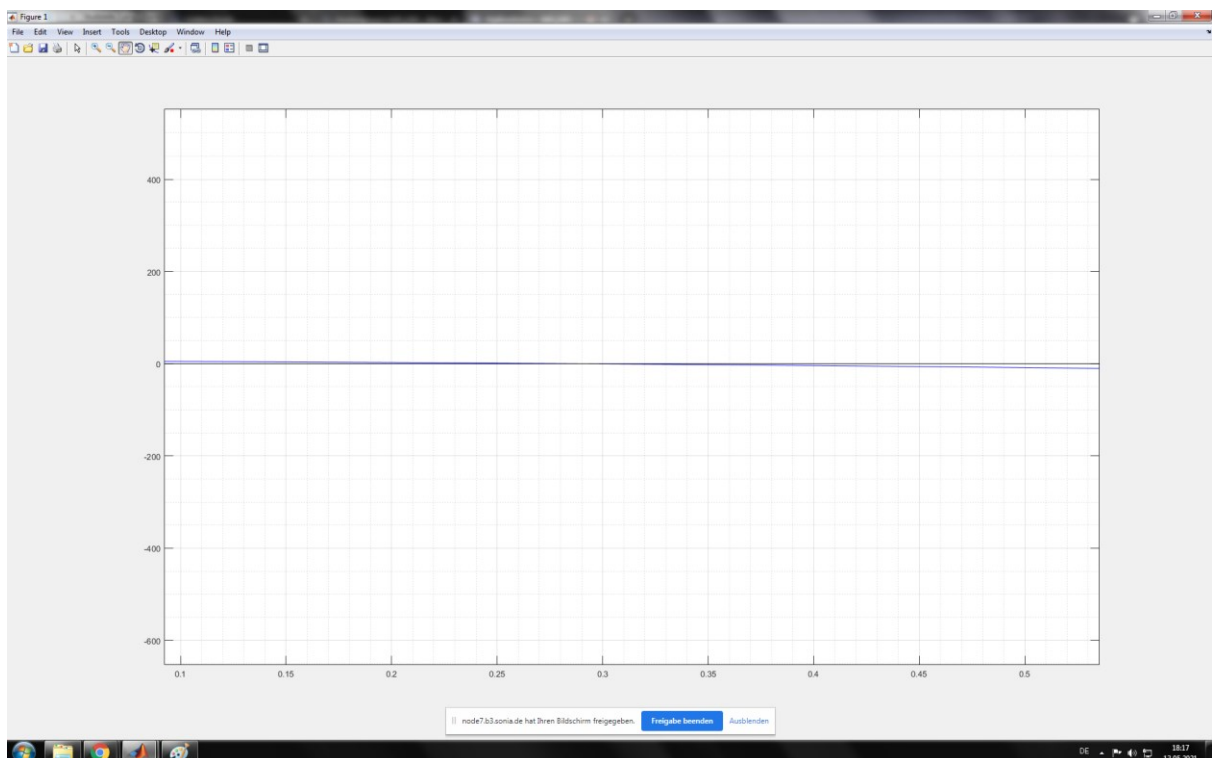
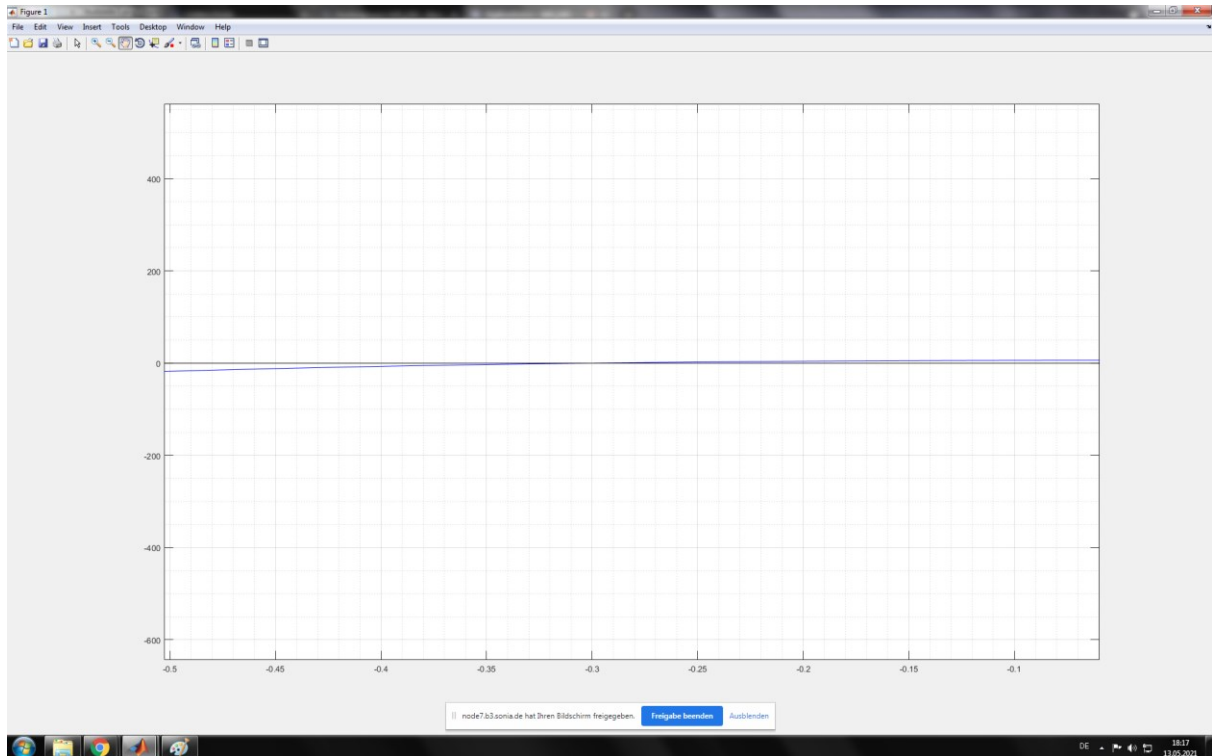


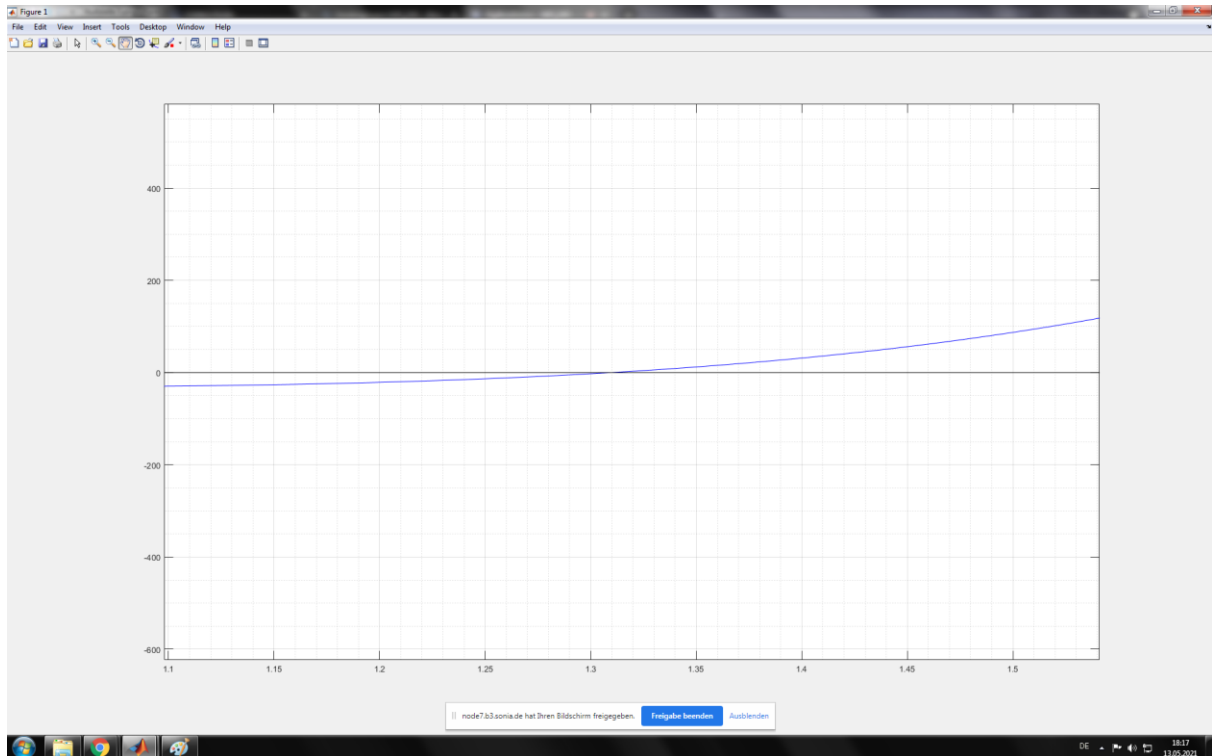
Aufgabe 4

Quelltext siehe *G08_H2_Aufgabe_4.pdf*

Indem Funktionsplot sind 4 Nullstellen zu erkennen:







Variables - aTABELLE							
aTABELLE							
	1	2	3	4	5	6	7
1 i	Xu	Xo	Xm	f(Xu)	f(Xm)	f(Xu)*f(Xm)	
2 0	-3.0000000000000000...	3.0000000000000000...	0.0000000000000000...	10545.000000000000...	6.0000000000000000...	>0	
3 1	0.0000000000000000...	3.0000000000000000...	1.5000000000000000...	6.0000000000000000...	87.1406250000000000...	>0	
4 2	1.5000000000000000...	3.0000000000000000...	2.2500000000000000...	87.1406250000000000...	2391.03735351562500...	>0	
5 3	2.2500000000000000...	3.0000000000000000...	2.6250000000000000...	2391.03735351562500...	6653.44568252563480...	>0	
6 4	2.6250000000000000...	3.0000000000000000...	2.8125000000000000...	6653.44568252563480...	10391.2557840943340...	>0	
7 5	2.8125000000000000...	3.0000000000000000...	2.9062500000000000...	10391.2557840943340...	12819.0321628907700...	>0	
8 6	2.9062500000000000...	3.0000000000000000...	2.9531250000000000...	12819.0321628907700...	14196.4805667202600...	>0	
9 7	2.9531250000000000...	3.0000000000000000...	2.9765625000000000...	14196.4805667202600...	14929.3657930129080...	>0	
10 8	2.9765625000000000...	3.0000000000000000...	2.9882812500000000...	14929.3657930129080...	15307.2766406210900...	>0	
11 9	2.9882812500000000...	3.0000000000000000...	2.9941406250000000...	15307.2766406210900...	15499.1538445215480...	>0	
12 10	2.9941406250000000...	3.0000000000000000...	2.9970703125000000...	15499.1538445215480...	15595.8298091396500...	>0	
13 11	2.9970703125000000...	3.0000000000000000...	2.9985351562500000...	15595.8298091396500...	15644.3530016920520...	>0	

Bei Grenzen -3 bis 3 liegen zu viele Nullstellen beieinander, sodass durch die Mittelwertbildung (x_m) Nullstellen "übersprungen" werden

- ➔ der Algorithmus hängt sich an der rechtsseitigen Grenze +3 auf, wo allerdings keine Nullstelle liegt
- ➔ Da $f(x_u) \cdot f(x_m)$ immer >0 denkt der Algorithmus die Nullstelle liegt rechtsseitig von x_m

Es gibt 4 Nullstellen im Bereich +3 bis -3 (siehe Abbildungen)

LÖSUNG DES PROBLEMS:

Variante 1:

- Graphen plotten
- Intervalle zur genauen Prüfung festlegen

Variante 2:

- Programm schreiben, dass in einer Schleife das große Intervall in viele kleine Intervalle zerlegt und das Bisektionsverfahren ausführt



Aufgabe 5

Quelltext siehe *G08_H2_Aufgabe_5.pdf*

```
C:\Users\koenigal.DIKAM\Desktop\HA2\HA2 Aufgabe 5\G08_H2_Aufgabe_5.m
2_Aufgabe_5.m
clear();
clc();
close all;                                     %Plots schließen

%KONFIGURATION-----
aLGS(1,:)=[ 2, 3, 8,-4,"X1", "=", 5];
aLGS(2,:)=[ 3,-2, 4,-8,"X2", "=", 8];
aLGS(3,:)=[ 3, 3, 5, 2,"X3", "=", 6];
aLGS(4,:)=[ 8, 9, 8, 6,"X4", "=", 9];
sKommastellenDouble = '%.8f';                 %Nachkommastellen
```

Variables - aX			
4x3 string			
	1	2	3
1	X1	=	1.43839541
2	X2	=	-1.55014326
3	X3	=	1.08022923
4	X4	=	0.46704871
5			
6			
7			
8			